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# Atomic GHZ States Prepared in Two Directly Coupled Cavities with Virtual Excitations in One Step*

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***Abstract*** *A scheme for one-step preparation of atomic GHZ states in two directly coupled cavities via virtual excitations is proposed. In the whole procedure, the information is carried only in two ground states of Λ-type atoms, while the excited states of atoms and cavity modes are virtually excited, leading the system to be insensitive to atomic spontaneous emission and photon loss.*

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## 1 Introduction

As an important resource for quantum information and computation, quantum entanglement attracts much attention.[1−3] In the last few years, most of researchers mainly focused on two-qubits quantum entanglement due to the experimental restriction.[4−5] But now, multiple-particles quantum entanglement with special forms (such as GHZ states, W states, and cluster states etc.) being more important in testing quantum nonlocality[6−7] and more secure in quantum communications,[8−9] becomes a subject of much interest.[10−12]

Cavity quantum electrodynamics (CQED) is thought to be an ideal candidate for the implementation of quantum information processing mainly due to the long life of atoms and the fast velocity of photons.[13] Thus, there have been lots of CQED-based works suggested for quantum-state preparation.[13−16] Recently, the coupled-cavities systems become the focus of CQED owing to its usefulness for distributed quantum communication and computation.[17−22] So far, a variety of technologies, such as microtoroidal cavity arrays,[23] microwave circuit cavities,[24] photonic crystal defects,[25] have been employed for realizing these systems. However, most of previous schemes used photons to carry (store) information, which may be sensitive to the photonic loss.

In Ref. [26], Zheng proposed a scheme for the generation of two-qubit entanglement with virtual excitations in two coupled cavities with an optical fiber connected. In the present protocol, we will show that this method can be generalized for the one-step preparation of GHZ states in two directly coupled cavities. The photons in cavities are only virtually excited and the information is only carried by the two ground states of Λ-type atoms so that the scheme is insensitive to the atomic spontaneous emission and decay of cavity fields.

## 2 Model of Multi-Atoms Interacting with Two Coupled Cavities

To begin with, we consider there are two directly coupled single-mode cavities (labeled by subscripts 1 and 2) and each one contains $N_j$ ($j = 1, 2$) 3-level atoms with an excited state $|f\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transition $|e\rangle \to |f\rangle$ for each atom in the $j$-th cavity is driven by a classical field with a detuning $\Delta_1$ and the Rabi frequency $\Omega_j$, while the transition $|g\rangle \to |f\rangle$ is coupled to the cavity mode with a detuning $\Delta_2$ and the coupling rate is $g_j$. The illustration for the system is shown in Fig. 1. The rotating-frame Hamiltonian describing the system is given by

$$H_r = -\Delta_1 \sum_{j=1}^{2}\sum_{k=1}^{N_j} |e\rangle_{jk}\langle e| - \Delta_2 \sum_{j=1}^{2}\sum_{k=1}^{N_j} |g\rangle_{jk}\langle g| + \Big[\sum_{j=1}^{2}\Omega_j \sum_{k=1}^{N_j} |f\rangle_k\langle e| + \sum_{j=1}^{2} g_j \sum_{k=1}^{N_j} |f\rangle_{jk}\langle g| a_j + \nu a_1^\dagger a_2 + \text{H.c.}\Big]\,, \tag{1}$$

where $a_j$ ($a_j^\dagger$) is the annihilation (creation) operator for the $j$-th cavity mode with frequency $\omega_c$, $\nu$ represents the hopping rate between two cavities and H.c. stands for Hermitian conjugate.

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In order to solve the system's evolution, let us introduce two bosonic operators $c_1 = (1/\sqrt{2})(a_1 + a_2)$ and $c_2 = (1/\sqrt{2})(a_1 - a_2)$.[18] The rotating-frame Hamiltonian is then rewritten as

$$H_r = -\Delta \sum_{j=1}^{2}\sum_{k=1}^{N_j} |e\rangle_{jk}\langle e| - (\Delta+\nu)\sum_{j=1}^{2}\sum_{k=1}^{N_j}|g\rangle_{jk}\langle g| + \nu(c_1^+ c_1 - c_2^+ c_2) + \Big[\sum_{j=1}^{2}\Omega_j \sum_{k=1}^{N_j}|f\rangle_{jk}\langle e| + \frac{1}{\sqrt{2}}g_1\sum_{k=1}^{N_1}|f\rangle_{1k}\langle g|(c_1+c_2) + \frac{1}{\sqrt{2}}g_2\sum_{k=1}^{N_2}|f\rangle_{2k}\langle g|(c_1-c_2) + \text{H.c.}\Big]\,. \tag{2}$$

And then, we turn to the interaction picture with respect to

$$H_0 = -\Delta_1\sum_{j=1}^{2}\sum_{k=1}^{N_j}|e\rangle_{jk}\langle e| - \Delta_2\sum_{j=1}^{2}\sum_{k=1}^{N_j}|g\rangle_{jk}\langle g| + \nu(c_1^+c_1 - c_2^+c_2)\,, \tag{3}$$

the interaction Hamiltonian for the system is given by the expression

$$H_I = \sum_{j=1}^{2}\Omega_j\sum_{k=1}^{N_j}|f\rangle_{jk}\langle e|\,\mathrm{e}^{\mathrm{i}\Delta_1 t} + \frac{1}{\sqrt{2}}\Big(g_1\sum_{k=1}^{N_1}|f\rangle_{1k}\langle g| + g_2\sum_{k=1}^{N_2}|f\rangle_{2k}\langle g|\Big)c_1\,\mathrm{e}^{\mathrm{i}(\Delta_2-\nu)t} + \frac{1}{\sqrt{2}}\Big(g_1\sum_{k=1}^{N_1}|f\rangle_{1k}\langle g| - g_2\sum_{k=1}^{N_2}|f\rangle_{2k}\langle g|\Big)c_2\,\mathrm{e}^{\mathrm{i}(\Delta_2+\nu)t} + \text{H.c.} \tag{4}$$

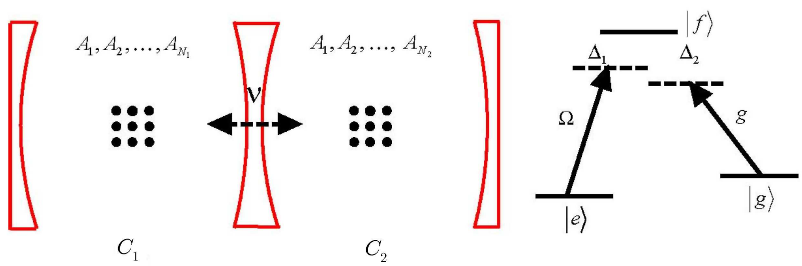


**Fig. 1** Schematic diagram for the one-step preparation of GHZ states and the transition of the three-level atoms.

For the sake of simplicity, we adjust the two detuning to satisfy $\Delta_1 = \Delta_2 - \nu = \Delta$. In the large detuning limit $\Delta\,, \nu, \Delta-\nu \gg g_j,\ \Omega_j$ and as long as the atoms are initialized in the ground states, the probability that the atomic excited states are populated is very small.[4] Then the effective Hamiltonian for the interaction Hamiltonian can be obtained using the method of Refs. [27–28], i.e.

$$H_{\text{eff}} = -\mathrm{i}H_I(t)\int H_I(t')\,\mathrm{d}t' = -\frac{1}{\Delta}\sum_{j=1}^{2}\Omega_j^2\sum_{k=1}^{N_j}|e\rangle_{jk}\langle e| - \frac{1}{2}\Big(\frac{1}{\Delta}c_1^+c_1 + \frac{1}{\Delta+2\nu}c_2^+c_2\Big)\sum_{j=1}^{2}g_j^2\sum_{k=1}^{N_j}|g\rangle_{jk}\langle g| - \frac{1}{\sqrt{2}\Delta}\sum_{j=1}^{2}\Omega_j g_j\Big(\sum_{k=1}^{N_j}|e\rangle_{jk}\langle g|c_1 + \sum_{k=1}^{N_j}|g\rangle_{jk}\langle e|c_1^+\Big)\,, \tag{5}$$

where we have neglected the high-order terms. In this Hamiltonian, the first and second terms represent the Stark shift for the state $|e\rangle$, induced by the classical fields and for the state $|g\rangle$, induced by cavities fields; the third term describes the Raman coupling, collectively induced by classical and cavities fields.

If the Rabi frequency $\Omega_j$ is much larger than the atom-cavity coupling $g_j$, i.e. $\Omega_j \gg g_l$ $(j,l = 1,2)$, the Stark shift for the state $|g\rangle_{jk}$ is very small relative to that of $|e\rangle_{jk}$ so that it can be neglected. Therefore, the effective Hamiltonian $H_{\text{eff}}$ reduces to

$$H'_{\text{eff}} = -\frac{1}{\Delta}\sum_{j=1}^{2}\Omega_j^2\sum_{k=1}^{N_j}|e\rangle_{jk}\langle e| - \frac{1}{\sqrt{2}\Delta}\sum_{j=1}^{2}\Omega_j g_j(A_j^+c_1 + A_j^-c_1^+)\,, \tag{6}$$

with $A_j^+ = \sum_{k=1}^{N_j}|e\rangle_{jk}\langle g|$ and $A_j^- = \sum_{k=1}^{N_j}|g\rangle_{jk}\langle e|$.

Now, let us further turn to the interaction frame with respective to $H'_0 = -(1/\Delta)\sum_{j=1}^{2}\Omega_j^2\sum_{k=1}^{N_j}|e\rangle_{jk}\langle e|$. The interaction

Hamiltonian for $H'_{\rm eff}$ is then given by

$$H'_{I,{\rm eff}} = -\frac{1}{\sqrt{2}\Delta}\sum_{j=1}^{2}\Omega_j g_j (A_j^+ c_1\,{\rm e}^{-{\rm i}\varepsilon_j t} + A_j^- c_1^+\,{\rm e}^{{\rm i}\varepsilon_j t})\,, \tag{7}$$

with $\varepsilon_j = \Omega_j^2/\Delta$. For simplicity, we set $\Omega_1 = \Omega_2 = \Omega$ and $g_1 = g_2 = g$ so that $\varepsilon_1 = \varepsilon_2$ ($\equiv \varepsilon \gg \Omega g/\sqrt{2}\Delta$) and the two cavities are virtually excited. The effective Hamiltonian described this situation becomes

$$\begin{aligned} H''_{I,{\rm eff}} &= -{\rm i}H'_{I,{\rm eff}}(t)\int H'_{I,{\rm eff}}(t'){\rm d}t' = \frac{\Omega^2 g^2}{2\varepsilon\Delta^2}[(A_1^- + A_2^-)(A_1^+ + A_2^+)c_1^+ c_1 - (A_1^+ + A_2^+)(A_1^- + A_2^-)c_1 c_1^+] \\ &= \frac{g^2}{2\Delta}[A^- A^+ c_1^+ c_1 - A^+ A^- c_1 c_1^+]\,, \end{aligned} \tag{8}$$

where $A^+ = A_1^+ + A_2^+ = \sum_{j=1}^{2}\sum_{k=1}^{N_j}|e\rangle_{jk}\langle g|$ and $A^- = (A^+)^+$.

If both of the two cavities fields are initially in the vacuum state, then the two bosonic fields are initially in the vacuum states. Thus, we obtain

$$H'''_{I,{\rm eff}} = -\eta A^+ A^-, \tag{9}$$

with $\eta = g^2/2\Delta$.

Combining the above calculations, we will obtain the atomic system state in the interaction time $t$, i.e.

$$\begin{aligned} |\psi(t)\rangle &= {\rm e}^{-{\rm i}H_0 t}\,{\rm e}^{-{\rm i}H'_0 t}\,{\rm e}^{-{\rm i}H'''_{I,{\rm eff}} t}|\psi(0)\rangle = \exp\Big\{{\rm i}t\Big[\Big(\Delta + \frac{\Omega^2}{\Delta}\Big)\sum_{k=1}^{N}|e\rangle_k\langle e| + (\Delta+\nu)\sum_{k=1}^{N}|g\rangle_k\langle g|\Big]\Big\}\exp(-{\rm i}H'''_{\rm eff}t)|\psi(0)\rangle \\ &= \exp\Big[{\rm i}t\Big(\frac{\Omega^2}{\Delta} - \nu\Big)\sum_{k=1}^{N}|e\rangle_k\langle e|\Big] \times \exp[{\rm i}\eta t A^+ A^-]\,|\psi(0)\rangle\,, \end{aligned} \tag{10}$$

where $|\psi(0)\rangle$ is the initial state of atoms. Here, we have eliminated the common phase ${\rm e}^{-{\rm i}(\Delta+\nu)t\left(\sum_{k=1}^{N}|e\rangle_k\langle e| + \sum_{k=1}^{N}|g\rangle_k\langle g|\right)}$. If we further choose $\Omega$, $\Delta$, and $\nu$ appropriately to satisfy

$$\frac{\Omega^2}{\Delta} - \nu = 0\,, \tag{11}$$

then

$$|\psi(t)\rangle = \exp[{\rm i}\eta t A^+ A^-]\,|\psi(0)\rangle\,. \tag{12}$$

So far, we have calculated the evolution of arbitrary atomic states with two cavities initially being in the vacuum states.

## 3 Generation of Atomic GHZ States

In this section, we will show that the above model can be applied to prepare atomic GHZ states trapped in two directly coupled cavities. We suppose that there are two directly coupled cavities with $N$ atoms distributed in both. The two cavities are initially in the vacuum state and each atom is prepared in the superposition state $(|e\rangle + |g\rangle)/\sqrt{2}$. So the initial state for the atomic system is[29]

$$|\psi(0)\rangle = \prod_{k=1}^{N}(|e\rangle_k + |g\rangle_k)/\sqrt{2} = 2^{-N/2}\sum_{m=0}^{N}(C_N^m)^{1/2}|m\rangle\,, \tag{13}$$

where $|m\rangle$ is a symmetric Dicke state with $m$ atoms in the excited state $|e\rangle$ and $N-m$ atoms in the ground states, i.e.

$$|m\rangle = (C_N^k)^{-1/2}\sum_{k=1}^{N}P_k(|e_1, e_2, \ldots, e_m, g_{m+1}, \ldots, g_N\rangle)\,, \tag{14}$$

where $\{P_k\}$ denotes the set of all distinct permutations of the qubits. It is noted that applying the collective raising and lowering operators to the symmetric Dicke state $|m\rangle$ will obtain

$$A^+|m\rangle = \sqrt{(m+1)(N-m)}|m+1\rangle\,, \quad A^-|m\rangle = \sqrt{m(N-m+1)}|m-1\rangle\,, \tag{15}$$

$$A^+A^-|m\rangle = m(N-m+1)|m\rangle\,. \tag{16}$$

According to Eq. (14), we can obtain the atomic system state $|\psi(t)\rangle$ at the interaction time $t$, i.e.

$$|\psi(t)\rangle = 2^{-N/2}\sum_{m=0}^{N}(C_N^m)^{1/2}\,{\rm e}^{{\rm i}m(N-m+1)\eta t}|m\rangle = 2^{-N/2}\sum_{m=0}^{N}(C_N^m)^{1/2}\,{\rm e}^{{\rm i}(N+1)m\eta t}\,{\rm e}^{-{\rm i}m^2\eta t}|m\rangle\,. \tag{17}$$

With the choice of $\eta t = \pi/2$, we have[29]

$$
\begin{aligned}
|\psi(\pi/2\eta)\rangle &= 2^{-N/2}\sum_{m=0}^{N}(C_N^m)^{1/2}\,\mathrm{e}^{\mathrm{i}(N+1)m\pi/2}(-\mathrm{i})^{m^2}|m\rangle \\
&= 2^{-(N/2+1)}\sum_{m=0}^{N}\big[\mathrm{e}^{-\mathrm{i}\pi/4}+\mathrm{e}^{\mathrm{i}\pi/4}(-1)^m\big](C_N^m)^{1/2}\,\mathrm{e}^{\mathrm{i}(N+1)m\pi/2}|m\rangle \\
&= \mathrm{e}^{-\mathrm{i}\pi/4}\Big[\prod_{k=1}^{N}(|e\rangle_k+\mathrm{e}^{\mathrm{i}\theta}|g\rangle_k)/\sqrt{2}+\mathrm{i}\prod_{k=1}^{N}(|e\rangle_k-\mathrm{e}^{-\mathrm{i}\theta}|g\rangle_k)/\sqrt{2}\Big]\,,
\end{aligned}
\tag{18}
$$

with $\theta = (N+1)\pi/2$. By far, we have prepared the GHZ states distributed in two coupled cavities.

## 4 Discussions and Conclusions

Now, let us make a brief discussion on the feasibility for the proposal. First, the 3-level used in our proposal is obtained from $^{87}$Cs, where the excited state $|f\rangle$, and two ground states $|e\rangle$ and $|g\rangle$ are represented by $|6P_{3/2}\,, F=4\rangle$, $|6S_{1/2}\,, F=3\rangle$, and $|6S_{1/2},\ F=4\rangle$, respectively.[30] The two directly coupled cavities can be two adjacent cavities[18] or two cavities with a short optical fiber connected.[31] We choose $\Delta_2 = 200g$, $\Delta_1 = \nu = 100g$, $\Omega = 10g$, and then the conditions $\Delta\,,\nu \gg \Omega \gg g$ and $\varepsilon \gg \Omega g/\sqrt{2}\Delta$ can easily be satisfied. In this situation, the interaction time $\tau$ required for GHZ generation is $\tau = \pi/2\eta = \Delta\pi/g^2 = 100\pi/g$. If we choose $g = 2\pi\times 110$ Mhz for atoms trapped in Fabry–Perot cavity, then $\tau \approx 0.45\ \mu$s, which is much smaller than the life of atomic ground state. Therefore, our scheme may be feasible based on current technologies.

In summary, we have proposed a scheme for the one-step preparation of GHZ states distributed in two directly coupled cavities. In the present proposal, only the atomic ground states are used for the storage of quantum information, leading to long storage time for the generated GHZ states. In addition, the atomic excited states and cavity modes are only virtually excited, so that the system is insensitive to atomic and photonic decay. Therefore, our scheme may be realized based on current technologies.

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